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Explanatory Notes on Geodetic Datums in Macao Special Administrative Region

1. Preface

Accompanying with changes in society and advancements in technology, the reception of Global Positioning System (GPS) was developed for civilian use for decades. The facility, swiftness, sophistication of GPS surveying, which is undoubtedly useful, can be applied in all-weather and various scales of sites without intervisibility among survey points. In recent years, traditional triangulation has gradually been replaced by the technology of GPS which also forges into a new era with the innovation of other GNSS systems (e.g. GLONASS, Galileo and Compass).

Nevertheless, coordinates of Macao Grid exercising on surveying and cartography are entirely different with that of World Geodetic System 84 (WGS84). Additionally, Macao coordinate system has linked with International Terrestrial Reference Frame (ITRF2005) since 2008. It is essential to illustrate the relationship among those coordinate systems. The following content will introduce each datum and coordinate system currently used in Macao, which enables users to obtain a better understanding and use them wisely.

2. Historical Background of Geodetic Datum in Macao Special Administrative Region

Five triangulation points were set up in Macao in 1907, while two were located on Avenida do Conselheiro Borja as a baseline, and one of them was defined with coordinates (0,0). Until 1919-1920, 60 survey points in Macao, Taipa and Coloane were undertaken a triangulation initially. The origin of Macao Grid renewed in 1943 by shifting its coordinates with 20000m to east and north respectively, from which the projected coordinate system in Macao was defined. Furthermore, astronomical coordinates of Macao projection origin were carried out through astronomical observations in 1912 and 1965 respectively. Macao geodetic network with 14 first order and nearly a hundred of secondary order triangulation control points were established by proceeding multiple times of surveying during 40s to 80s of the 20th century.

Until 1981, 512 Specialist Team Royal Engineers (STRE) of the U.K. Military

Survey utilized GPS surveying to establish coordinates of WGS72 reference ellipsoid for two triangulation control points: Monte de Barra and Coloane Alto, which registered globally with codes No.31386 and No.31397 respectively in the linkage with Hong Kong Doppler stations. It is a milestone of GPS application in Macao.

In 1990, 512 STRE engineers re-undertaken GPS surveying in Macao to establish the WGS 84 reference datum by GPS joint survey and adjustment of triangulation points in Macao and control points in Hong Kong. Apart from the points at Monte da Barra and Coloane Alto, four more points were set up in Macau Peninsula and the islands of Taipa and Coloane, thus there were totally six Doppler stations constituting the initial GPS control network in Macao.

In 1994, Macao Cartography and Cadastre Bureau extended the GPS network to 14 first order and several secondary order control points, and acquired a set of transformation parameters between Macao Grid and WGS84 (it is different from the genuine coordinates of WGS84 by using coordinates of Doppler station as a datum). This is an essential foundation to establish GPS application hereafter.

As Continuously Operating Reference Stations (CORS) are popularized around the world in the 21st century, Macao Cartography and Cadastre Bureau has also met the actual needs, set up three satellite positioning reference stations at Monte Fortress, Coloane Alto and Taipa Grande in 2002, 2006 and 2008 respectively in order to collect GPS data continuously in all-weather and provide RTK information. By using the collected data in 2008, a joint solution with 10 IGS (International GNSS Service) stations situated near Macao was done to obtain coordinates of the three reference stations under ITRF2005. In the following year, 2009, ITRF2005 was extended to the Macao first order control points and the relationship with the Macao Grid was also established.

For the elevation datum, the elevations shown in topographic map and the altitude in Macao refers to the mean sea level, which is confirmed by the data collected from the tide gauge station at Má Káu Séak in 40 years between 1925 and 1964 and has been hitherto used ever since. The Macao elevation datum was originally referenced to the benchmark at Má Káu Séak, later on, the tidal data was recorded by the tide gauge situated at Inner Harbor since 1984. And the current original benchmark of Macao elevation system is located in front of the A-Ma Temple .

3. Geodetic Datum and Coordinate Systems in Macao

3.1 Macao Grid System

Macao Grid System, which is a projected coordinate system, adopts the reference ellipsoid of the International Hayford 1924, and the projection is Transverse Mercator Projection. Its projection parameters are as follows:

Reference Ellipsoid	International Hayford a (Major semi-axis)=6378388.0 m f (Flattening)=1/297.0
Projection Method	Transverse Mercator Projection
False Easting	20000.00 m
False Northing	20000.00 m
Latitude of Origin	22° 12′ 44.6300″ N
Longitude of Origin	113° 32′ 11.2900″ E
Scale factor	1

Table 1

3.2 World Geodetic System 84 (WGS 84)

From WGS60, WGS66, WGS74 to the current WGS84, WGS, a global earth-centered earth-fixed coordinate system, set up by Department of Defense of the United States since 1980s. Its origin is the center of earth mass, while its z-axis points to the terrestrial pole (CTP) defined by IERS (International Earth Rotation Service), x-axis to the intersection of meridian plane in zero degree and CTP Equator. Y-axis constitutes a right-handed coordinate system with z and x axes. The fundamental parameters of its ellipsoid are as follows:

Major semi-axis (a)	6378137.0 m
Flattening (*f*)	1/298.257223563

Table 2

3.3 International Terrestrial Reference Frame (ITRF2005)

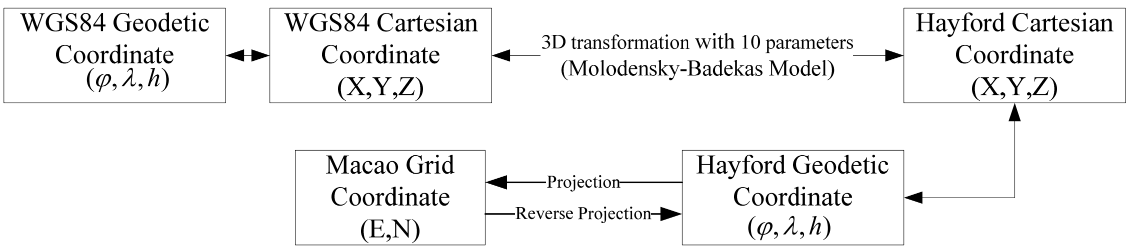
ITRF (International Terrestrial Reference Frame), provided by IERS, is an international accepted reference frame with the highest preciosity and stability. Its dynamic global reference frame is composed of various observational technologies and several analytic centres in order to satisfy the studies on regional and universal global variation, such as changes of the mean sea level, regional settlement, crustal movement, plate deformation etc. ITRF2005, issued in 2006, can be deemed as coincident with WGS84 (G1150) [1], which has a difference in the range of 1-2 cm. The parameters of GRS1980, the ellipsoid of ITRF, are as follows:

Major semi-axis (a)	6378137.0 m
Flattening (*f*)	1/298.257222101

Table 3

4. Coordinate Transformation

There is a coordinate transformation and mapping projection between Macao Grid (a plane Cartesian coordinate system) and WGS84/ITRF2005 (an ellipsoidal geodetic coordinate system). A set of 3D transformation model has been used in Macao since 1994, its process is as follows:

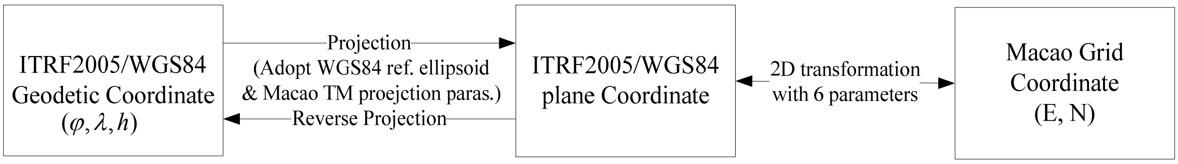


Based on the flow chart above, it needs two sets of coordinates (X,Y,Z or φ, λ, h) in WGS84 and Hayford of common points to figure out 3D transformation parameters. In practical application, 3D coordinates (geodetic latitude φ, geodetic longitude λ and geodetic height h) of WGS84 can be obtained directly through GNSS. However, genuine 3D geodetic coordinates cannot be figured out through reverse projection algorithm from traditional geodetic surveying, it is essentially the coordinate system of 2D+1D. The datum of plane coordinate system is a Hayford reference ellipsoid, however, height is the mean sea level instead of a genuine geodetic height, it will lead

[1] **US National Geospatial Intelligence Agency (NGA)**, *Addendum to NIMA TR 8350.2: Implementation of the World Geodetic System 1984 (WGS 84) Reference Frame G1150*, 2003.

to a height deviation of Cartesian coordinate system during transformation.

Therefore, a set of transformation parameters ought to reform by connecting the Macao Grid and ITRF2005 in 2009, and in introducing a relation of the coordinate transformation 2D+1D, plane coordinates use the 2D transformation with six parameters (Appendix 1), while height is the distance from the surface of the reference ellipsoid to the geoid or mean sea level. In this way, leveling heights can be converted into geodetic heights (refer to 5.). The process of 2D transformation is as follows:



Coordinate transformation in 2D+1D is more rational than that in 3D, by which the converted coordinates are not affected by a deviation in the geodetic height, but it would be inconvenient to deal with plane coordinates and heights individually. Therefore, we also offer 3D transformation parameters between ITRF2005 and Macao Grid (Appendix 2) in order to cater for different users.

5. Transformation of Elevation Datums

Macao elevation system adopts the height of the mean sea level, this height of leveling (H_L) is the distance from the ground to the surface of the mean sea level, while the height of WGS (H_{WGS}) is the normal distance of a reference ellipsoid from the ground to the surface of the reference ellipsoid. The two datums have a difference ($H_L = H_{WGS} - diff$) which varies irregularly with locations and can generate a model in accordance with mathematical measures. The method is to adopt heights of some well-distributed common points in leveling and WGS84 and a polynomial of height fitting model in order to figure out the relevant parameters (Appendix 3). Points with ellipsoidal heights acquired by GNSS observation can be converted to leveling heights by calculating the height differences using the known parameters and the above formula.

6. Examples of Coordinate Transformation

2D Transformation		1	2	3
ITRF2005 Geodetic Coordinates	Latitude	22° 11′ 40.000″ N	22° 09′ 30.000″ N	22° 07′ 20.000″ N
	Longitude	113° 32′ 50.000″ E	113° 32′ 50.000″ E	113° 34′ 50.000″ E
	Ellipsoidal Height (m)	10.00	20.00	30.00
ITRF2005 Projected Coordinates	Easting (m)	21108.83	21109.12	24548.52
	Northing (m)	18012.07	14013.39	10015.35
Macao Grid Coordinates	Easting (m)	20800.08	20802.10	24243.21
	Northing (m)	18145.04	14146.39	10149.87

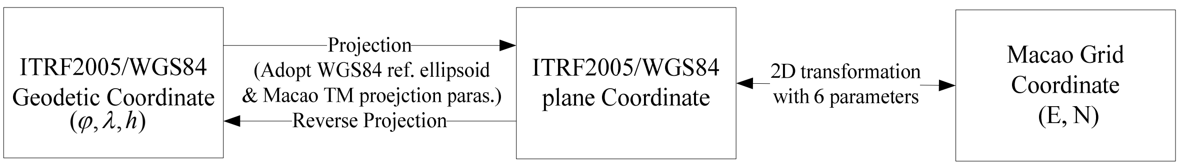
3D Transformation		1	2	3
ITRF2005 Geodetic Coordinates	Latitude	22° 11′ 40.000″ N	22° 09′ 30.000″ N	22° 07′ 20.000″ N
	Longitude	113° 32′ 50.000″ E	113° 32′ 50.000″ E	113° 34′ 50.000″ E
	Ellipsoidal Height (m)	10.00	20.00	30.00
ITRF2005 Cartesian Coordinates	X (m)	-2360431.93	-2361038.62	-2364796.74
	Y (m)	5416409.60	5417801.75	5417816.89
	Z (m)	2394366.28	2390667.16	2386967.10
Hayford Cartesian Coordinates	X (m)	-2360227.87	-2360836.14	-2364595.60
	Y (m)	5416714.29	5418105.72	5418119.66
	Z (m)	2394521.78	2390822.68	2387124.02
Hayford Geodetic Coordinates	Latitude	22° 11′ 44.325″ N	22° 09′ 34.327″ N	22° 07′ 24.381″ N
	Longitude	113° 32′ 39.220″ E	113° 32′ 39.286″ E	113° 34′ 39.342″ E
	Ellipsoid Height (m)	13.89	23.79	33.54
Macao Grid Coordinates	Easting (m)	20800.08	20802.10	24243.21
	Northing (m)	18145.04	14146.39	10149.87
	Height of Mean Sea Level (m)	13.89	23.79	33.54

Transformation of GNSS Elevation Fitting		1	2	3
Macao Grid Coordinates	Easting (m)	20800.08	20802.10	24243.21
	Northing (m)	18145.04	14146.39	10149.87
ITRF2005 Geodetic Height	Ellipsoid Height (m)	10.00	20.00	30.00
Height of Mean Sea Level	Height of Fitting Model(m)	13.88	23.78	33.54

Appendix 1: Transformation of Geodetic Datums in Macao and Data of Map Projection

2D Transformation between ITRF2005/WGS84 and Macao Grid

Macao Cartography and Cadastre Bureau connected Macao coordinate system to ITRF2005 by using the three Macao satellite positioning reference stations in 2008. The reference time was set on 17th May 2008 at noon (the epoch: 2008.37568). The relationship of 2D coordinate transformation between ITRF2005 and Macao Grid is as follows:



Parameters of Transverse Mercator Projection in Macao	
Projection	Transverse Mercator (TM) Projection
False Easting	20000.00m
False Northing	20000.00m
Latitude of Origin	22°12′ 44.6300″ N
Longitude of Origin	113°32′11.2900″ E
Geoid	Mean Sea Level
Scale Factor	1

Formula of 2D Transformation with 6 Parameters:

$$\begin{bmatrix} E_2 \\ N_2 \end{bmatrix} = \begin{bmatrix} dE \\ dN \end{bmatrix} + \begin{bmatrix} E_0 \\ N_0 \end{bmatrix} + (1+m)\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} E_1 - E_0 \\ N_1 - N_0 \end{bmatrix}$$

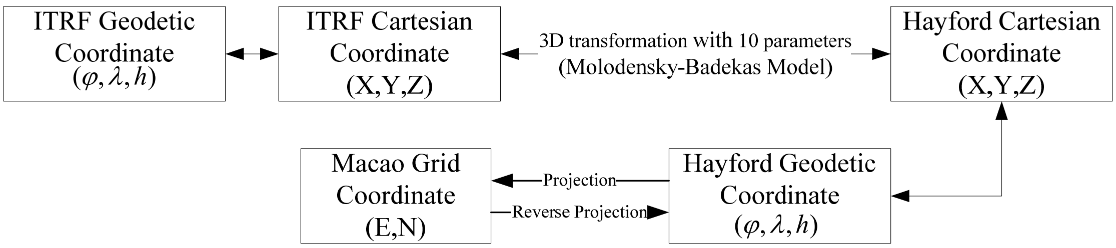
[translation parameters (*dE, dN*), rotation parameter (α), scale parameter (m), origin of rotation (E_0, N_0), Cartesian coordinates of source coordinate system (E_1, N_1), Cartesian coordinates of target coordinate system (E_2, N_2)]

2D Transformation with 6 parameters		
	From：ITRF2005/WGS84 To：Macao Grid	From：Macao Grid To：ITRF2005/WGS84
Origin of Rotation	E_0：21995.742m N_0：14829.896m	E_0：21688.365m N_0：14963.270m
Shift along Easting	-307.377m	307.377m
Shift along Northing	133.374m	-133.374m
Rotation Amount	-1′ 29.586″	1′ 29.586″
Scale Factor	-6.513 ppm	6.513 ppm

Appendix 2: Transformation of Geodetic Datums in Macao and Data of Map Projection

3D Transformation between ITRF2005/WGS84 and Macao Grid

Macao Cartography and Cadastre Bureau connected Macao coordinate system to ITRF2005 by using the three Macao satellite positioning reference stations in 2008. The reference time was set on 17th May 2008 at noon (the epoch: 2008.37568). The relationship of 3D coordinate transformation between ITRF2005 and Macao Grid is as follows:



Formula of 3D Transformation with 10 Parameters:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} + (1+m)\cdot R \cdot \begin{bmatrix} X_1 - X_0 \\ Y_1 - Y_0 \\ Z_1 - Z_0 \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}$$

$$R(\alpha,\beta,\gamma) = \begin{pmatrix} \cos\beta\cos\gamma & \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma & \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma \\ -\cos\beta\sin\gamma & \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma \\ \sin\beta & -\sin\alpha\cos\beta & \cos\alpha\cos\beta \end{pmatrix}$$

[translation parameters (dX, dY, dZ), rotation parameters of X,Y and Z axes (α, β, φ), scale parameter (m), origin of rotation (X_0, Y_0, Z_0), Cartesian coordinates of source coordinate system (X_1, Y_1, Z_1), Cartesian coordinates of target coordinate system (X_2, Y_2, Z_2)]

3D Transformation with 10 Parameters		
	From：ITRF2005 To：Macao Grid	From：Macao Grid To：ITRF2005
Reference Ellipsoid A	GRS 1980	International Hayford
Reference Ellipsoid B	International Hayford	GRS 1980
Transformation Mode	Molodensky-Badekas	Molodensky-Badekas
Origin of Rotation	X_0：-2361757.652 m Y_0：5417232.187 m Z_0：2391453.053m	X_0：-2361554.788 m Y_0：5417536.177 m Z_0：2391608.926 m
Shift along x-axis	202.865 m	-202.865 m
Shift along y-axis	303.990 m	-303.990 m
Shift along z-axis	155.873 m	-155.873 m
Rotation about x-axis	34.067″	-34.079″
Rotation about y-axis	-76.126″	76.126″
Rotation about z-axis	-32.647″	32.660″
Scale Factor	-6.096 ppm	6.096 ppm

Parameters of Transverse Mercator Projection in Macao	
Projection	Transverse Mercator (TM) Projection
False Easting	20000.00m
False Northing	20000.00m
Latitude of Origin	22°12′44.6300″ N
Longitude of Origin	113°32′11.2900″ E
Geoid	Mean Sea Level
Scale factor	1

Appendix 3: Height Transformation between Leveling and WGS84

Formula between Height of Leveling(H_L) and Height of WGS84(H_{WGS})：
$H_L = H_{WGS} - diff$ ------ (1)

Polynomial of Elevation Fitting Formula:
$diff_i = a_1 + a_2E_i + a_3N_i + a_4 E_i^2 + a_5 E_i N_i + a_6 N_i^2$ ------ (2)

Parameters are solved by using the observations at common points:

a_1=	-5.1810704571	a_4=	-0.0000000017
a_2=	0.0001223073	a_5=	-0.0000000007
a_3=	-0.0000163659	a_6=	0.0000000001

Example:

WGS84 latitude, longitude and ellipsoidal height (22° 11′ 40.000″ N, 113° 32′ 50.000″ E, 10.00 m) are acquired respectively by GNSS observation, and are transformed into Macao Grid E, N (20800.082 m, 18145.042 m) by using a projection and the 2D transformation with 6 parameters.

Substitute into the above formula(2), diff=-3.88m as the result, then substitute into formula(1), H_L = 13.88 m as the result.